DEPENDENCE OF LOCAL INTERACTION PARAMETERS ON THE KNUDSEN NUMBER

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16. Abstract

The drag coefficients of a circular cylinder, a sphere, and a cone are calculated on the basis of an interpolation formula over the entire range of Knudsen numbers. The analytical values are compared with drag data obtained in a low-density wind tunnel at Mach numbers greater than 4.

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Experiments in low-density wind tunnels at Mach numbers $M \geqslant 4$ [Refs. 1-3] show that the drag coefficients C_d of a sphere and cylinder in the entire range of change of the Knudsen number are adequately approximated by the interpolation formula /180

$$C_D = C_{Dm}\beta + C_{Dk}(1-\beta), \tag{1}$$

where subscripts m and k above and hereinafter denote quantities pertaining to the free molecular (Kn = ∞) and continuum (Kn = 0) limits, and

$$\beta = \Phi\left(\frac{\lg \mathrm{Kn} - a}{\sigma}\right), \qquad \Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}\,dy. \tag{2}$$

In Eq. (2), the parameter a is chosen on the basis of the condition that $\beta = 1/2$ when $\mathrm{Kn} = 10^2$, and σ is the spread of the tran[illegible] on zone between Kn = ∞ and Kn = 0 (to an accuracy of 0.001, C_D(Kn) changes only when $-3\sigma \leqslant \log \mathrm{Kn} \leqslant 3\sigma$). Since the parameters a, σ represent the flow regime, they should be expected to be weakly dependent on the geometry of the body in the flow. This is confirmed by the following hypothesis.

<u>Theorem</u>. The Knudsen number is the ratio of the mean free path λ of atoms to the so-called "wet length" $\sqrt{S_+}$, where S_+ is the area of the illuminated portion of the body in the flow, i. e.,

$$\mathrm{Kn}_0 = \lambda/\sqrt{S_+}, \quad S_+ = \iint_{\cos\theta \geq 0} dS, \quad \theta = \angle(\bar{n}, -\bar{v}),$$

$\bar{n}$ is the outer normal to the surface S of the body, and $\bar{v}$ is the flow velocity. Then in Eq. 2, the parameters a and σ are the same, at least for a sphere, cylinder and acute cones with half-aperture angles δ = 5°, 10° and 25°.

Figure 1 shows function [2], in which σ = 0.77, and α is a Gaussian random quantity with average <a> = -0.878 and dispersion s^2 = 0.0676. The middle curve in this figure was plotted for a_0 = <a> = -0.878, and the extreme ones, for

$$a_0^- = \langle a \rangle - 1{,}96\ s = -1{,}388 \quad \text{(upper)}$$
$$a_0^+ = \langle a \rangle + 1{,}96\ s = -0{,}368 \quad \text{(lower)}$$

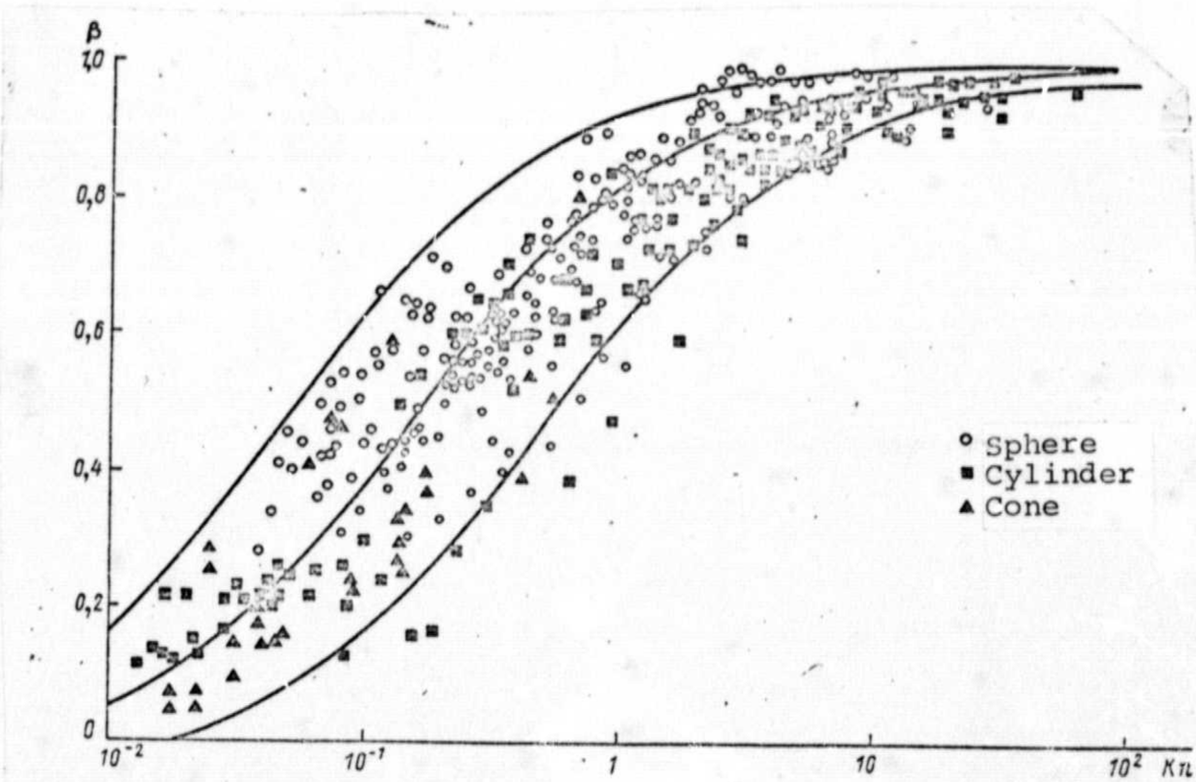

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Fig. 1. Comparison of theroretical and experimental values of β for a sphere, a cylinder and a cone.

We see that the experimental points, taken for the sphere and cylinder of Ref. [3], and for cones of Ref. (4), cluster around the middle curve within 95% of the confidence interval bounded by the extreme curves. In particular, for a sphere, of the total number* of points, 202 points lie within the range, i. e., 96%, and for a cylinder, 132 points out of 139 lie within this range, i. e., 95%. The spread of the experimental points is explained by the neglect of other factors in the experiment, in particular, the temperature factor. /182

It is interesting to note that $\langle a \rangle = -0.878$ was calculated from the experimental C_D curve of a sphere, obtained by Koppenwallner[1] (Fig. 2). Figure 3 demonstrated that this curve is recalculated exactly from formula (1) for a cylinder placed across the flow if the diameter of the cylinder** is taken as characteristic dimension. The experimental points in Fig. 3 are also due to Koppenwallner[2]. Finally, Fig. 4 shows a curve for cones with half-aperture angles $\delta = 5°$, $10°$ and $25°$ for β calculated from formula (2) with the parameters $a = \langle a \rangle = -0.878$ and $\sigma = 0.77$, $C_{Dk}^{k} = 0.016, 0.062, 0.35$ for δ $\delta = 5$, 10 and 25° respectively, $C_{Dk}^{k} = 2.23$ for $\delta = 5°$, $10°$ and $C_{Dk}^{k} = 2.62$ for $\delta = 25°$. In Figs. 2-4, $Kn = \lambda/2R$, where R is the radius of a sphere, cylinder, or base of a cone.

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*Translator's note: The actual number is missing from the Xerox copy of the Russian original.

**The problem of flow around a sufficiently long cylinder across the flow in the middle portion, where the influence of the ends is not manifested, may be treated as a planar problem, i. e., the characteristics dimension is the diameter.

This number Kn is related to $Kn_0 = \lambda/\sqrt{S_+}$ by the obvious formulas:

$$Kn = Kn_0 \sqrt{2\pi}/2 \quad \text{for a sphere,}$$
$$Kn = Kn_0 \sqrt{\pi}/2 \quad \text{for a cylinder of length R and radius R,}$$
$$Kn = Kn_0 \sqrt{\pi}/2\sqrt{\sin\delta} \quad \text{for a cone,}$$

since for a sphere $S_+ = 2\pi R^2$, for a cylinder $S_+ = \pi R^2$, and for a cone $S = \pi R^2/\sin\delta$.

As is evident from Figs. 1-4, the statement of the theorem is beautifully confirmed by the experiment for such different bodies as a sphere, cylinder and cone. It is reasonable therefore th assume that this statement is valid in the general case as well.

Using the theorem, we will now obtain the dependence of the local interation parameters on the Kn at M > 4.

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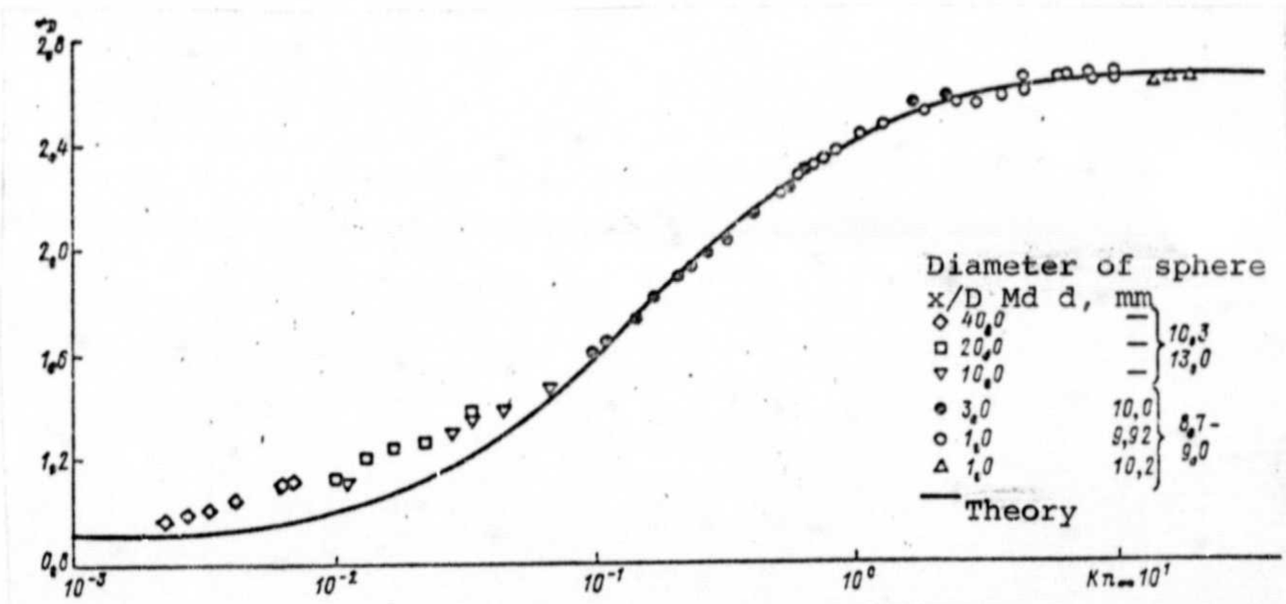

Fig. 2. Drag coefficient of a sphere in air.

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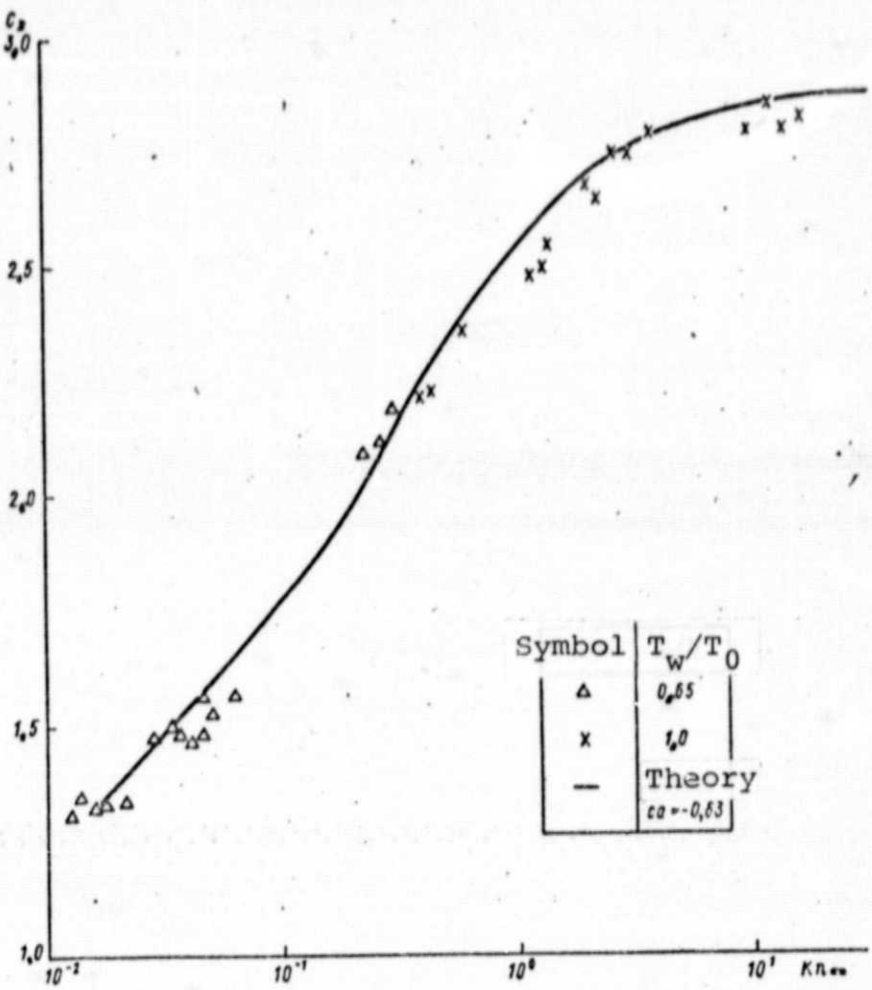

Fig. 3. Drag coefficient of a cylinder in air.

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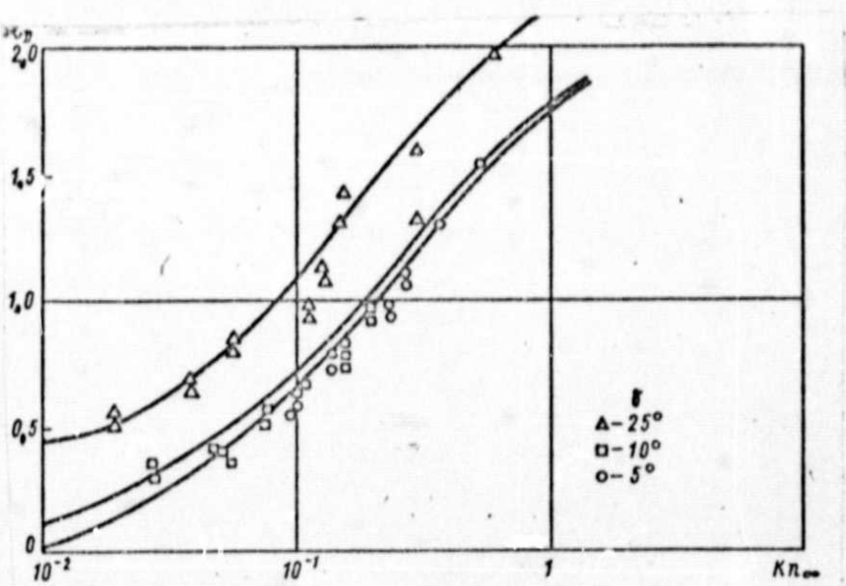

Fig. 4. Drag coefficient of a cone in air.

We take the thre-parameter scheme discussed in Refs. [5-8], when the local pulse flow in the surface of a body referred to $1/2 \cdot \varrho v^2$, is expressed by the formulas

$$\left.\begin{array}{l} \bar{p} = \begin{cases} -p(\theta)\bar{n} - \tau(\theta)\bar{t}, & \cos\theta \geq 0, \\ 0, & \cos\theta < 0, \end{cases} \\ p(\theta) = \mu\cos\theta + (\lambda - 3\lambda_2)\cos^2\theta - \mu\cos^3\theta + 4\lambda_2\cos^4\theta, \\ \tau(\theta) = (\lambda_1 - 3\lambda_2)\cos\theta - \mu\cos^2\theta + 4\lambda_2\cos^3\theta, \end{array}\right\} \tag{3}$$

where ϱ is the gas density.

As we know[5], in this case for axisymmetric convex bodies, /186 the drag coefficients referred to $1/2 \cdot \varrho v^2 \pi R^2$ for the force and to $1/2 \cdot \rho v^2 \pi R^2 L$ (L being the length of the body) for the moment, are

$$C_x(\alpha) = \lambda_1 C_{x1}(\alpha) + \lambda_2 C_{x2}(\alpha),$$
$$C_y(\alpha) = \mu C_{y1}(\alpha), \quad C_{mz} = \lambda_1 C_{m1} + \lambda_2 C_{m2} + \mu_3 C_{m3}, \tag{4}$$

where C_{x1}, C_{x2}, C_{y1}, C_{m1}, C_{m2}, C_{m3} are functions of the shape of the body in the flow and of the angle of attack (so-called shape factors), and λ_1, λ and μ depend solely on the flow regime and are independent[7,8] of the shape of the body (so-called local interaction parameters). The shape factors for typical bodies can be calculated once and for all,[5] so that the problem of determination of the forces and moments in a transient regime reduces to finding the dependence of λ_1, λ_2 and μ on the parameters of the regime.

We will denote any λ_1 and λ_2 by ξ_1 and, in accordance with the theorem, represent ξ_1 in the form

$$\xi_1 = \xi_\text{н}\beta + \xi_\text{к}(1-\beta). \tag{5}$$

Having the free molecular values of ξ_m and values of ξ_k at the boundary of a continuous medium, we find ξ_1 from formula (5) for any value of Kn_0 of the transition zone. Formula (4) makes it possible to determine the maximum ξ_m and ξ_k, easily if C_x and C_y are known for the same angle of attack of two bodies of different shapes, for example, a sphere and a cylinder. For a sphere of unit radius

$$C_{x1}^{c}=1, \quad -C_{x2}^{c}=1; \tag{6}$$

for a cylinder of radius R = 1 and length R = 1

$$C_{x1}^{u}=1, \quad C_{x2}^{u}=-1/3, \tag{7}$$

so that by solving the system of equations

$$\left.\begin{aligned} C_D^{c} &= \lambda_1 C_{x1}^{c} + \lambda_2 C_{x2}^{c}, \\ C_D^{u} &= \lambda_1 C_{x1}^{u} + \lambda_2 C_{x2}^{u}, \end{aligned}\right\} \tag{8}$$

with the above-indicated values of C_{x1} and C_{x2}, we find /187
λ_1 and λ_2. In particular, in a continuum regime, Kn = 0 at large M, and Newton's formula [9] is used to find C_x, C_y:

$$p=\begin{cases} 2p_0\cos^2\theta, & 0\leq\theta\leq\pi/2, \quad \tau=0, \\ 0, & \pi/2<\theta\leq\pi/2. \end{cases} \tag{9}$$

where

$$p_0=\frac{1}{\varkappa M^2}\left[\left(\frac{\varkappa+1}{2}M^2\right)^{\frac{\varkappa}{\varkappa+1}}\left(\frac{\varkappa+1}{2\varkappa M^2-\varkappa+1}\right)^{\frac{1}{\varkappa-1}}-1\right]$$

$$\lim_{M\to\infty} p_0=\frac{1}{\varkappa}\left(\frac{\varkappa+1}{2}\right)^{\frac{\varkappa}{\varkappa+1}}\left(\frac{\varkappa+1}{2\varkappa}\right)^{\frac{1}{\varkappa-1}}. \tag{10}$$

Formulas (9)-(10) lead to the equalities

$$C_{D\kappa}^{c}=p_0, \quad C_{D\kappa}^{u}=4/3\,p_0, \tag{11}$$

and for a cone with a half-aperture [9] angle δ

$$C_{D\kappa}^{\kappa}=\left[2\,tg^2\delta/(1+tg^2\delta)\right]p_0.$$

We find from (5)-(6) and (11) that

$$\lambda_{1\kappa}=3/2\,p_0, \quad \lambda_{2\kappa}=1/2\,p_0.$$

A similar method is used to find λ_{1m}, λ_{2m}, and for C^{C}_{Dm} and C^{u}_{Dm}, theoretical solutions of problems of free molecular flow as well as experimental values can be taken. While the number M entered into the parameters λ_1 and λ_2 via λ_{1m} and λ_{2m}, in addition to M, the parameters of interaction of the gas atoms with the wall, including the temperature factor, will enter via λ_{1m} and λ_{2m}. Therefore, the locality hypothesis and formulas (4)-(5) represent an extensive area of activity for solving the reverse problems: from known C_x, C_y, C_{mz} to reconstruct the regime, parameters M, Kn, etc.

Figures 5-6 demonstrate the type of dependence of λ_1, λ_2 and μ on Kn when = 1.4, M = ∞ and

$$\lambda_{1м} = 2{,}94, \quad \lambda_{2м} = 0{,}24, \quad \mu'_м = 1{,}25. \qquad (12)$$

The values (12) are the solution of system (8) for a sphere and a cylinder with the following values obtained in the funnel:[1-2] $C^{c}_{Dm} = 2.7$, $C^{u}_{Dm} = 2.86$.

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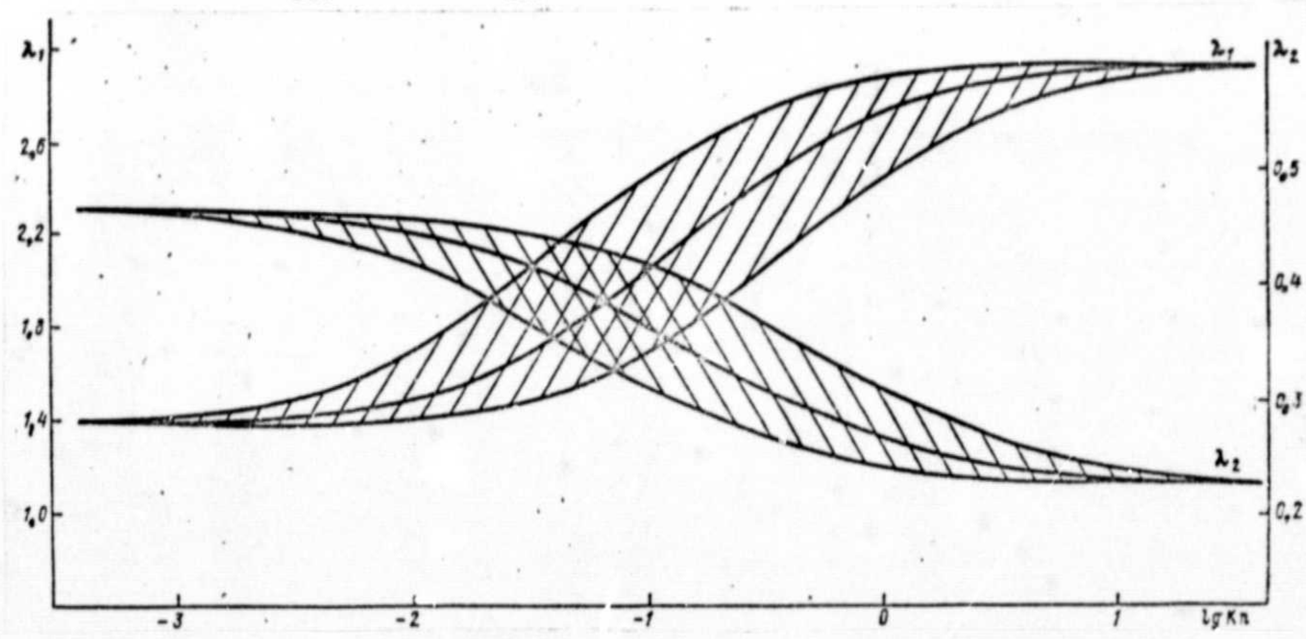

Fig. 5. The functions $\lambda_1 = \lambda_1(\text{Kn})$, $\lambda_2 = \lambda_2(\text{Kn})$ in air, M > 4.

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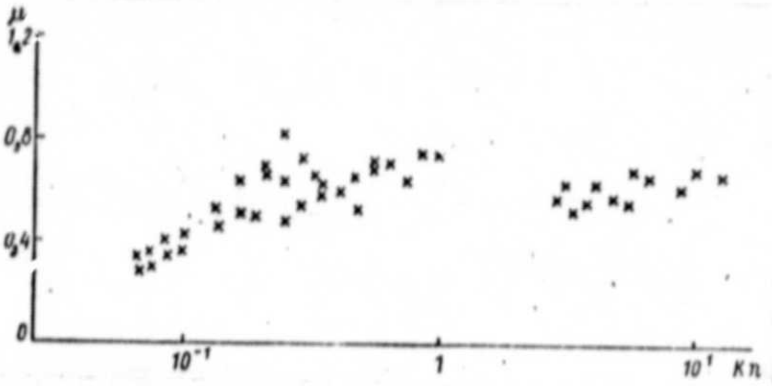

Fig. 6. The functions $\mu = \mu(Kn)$, M = 4-5.

The value of μ was determined from V. Ya. Ponomarev's experiment on the flow aroung an inclined cylinder [10] to be $\mu = C_y(\alpha)/C_{y1}(\alpha)$. Since the function $C_y(Kn)$ is nonmonotonic, [11] $\mu(Kn)$ is not expressed by formula (5). Unfortunately, it is not possible to represent this dunction reliably because of insufficient experimental data for C_y in the entire range of Knudsen numbers. The shape of the $\mu(Kn)$ curve is the same as the shape of C_y cones in [Ref. 11].

The middle curves in Fig. 5 were calculated for $a = \langle a \rangle = -0.878$, and the extreme curves, for (3). The shaded areas correspond to a 96% confidence interval.

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